Peter Lee on September 29, 2009

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3:40 PM

Given a bialgebra (BA, QTBA, QTHA, ...) A,

Rep(A): Category of reps. of A.

A "monoidal" category.

BA $\rightsquigarrow$ Monoidal category

QTBA $\rightsquigarrow$ Braided monoidal category.

QTHA $\rightsquigarrow$ —"— with duals for objects.

Monoidal category: a category $\mathcal{C}$ with a bifunctor $\otimes : \mathcal{C} \times \mathcal{C} \to \mathcal{C}$ which is "quasi-associative":

$\exists\ \Phi_{x,y,z} : (x \otimes y) \otimes z \to x \otimes (y \otimes z)$ (Φ is natural)

for x, y, z objects of $\mathcal{C}$

$\exists$ a unit 1 for $\otimes$, up to a natural iso:

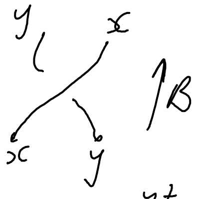
$l_x : 1 \otimes x \xrightarrow{\sim} x \quad r_x : x \otimes 1 \xrightarrow{\sim} x$

with a pentagon & triangles for the units.

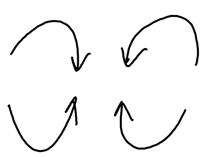
$M, N \in$ Rep(A), $a \in A$, $m \in M$, $n \in N$

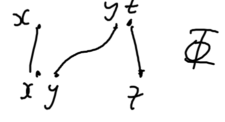
$a(m \otimes n) := (\Delta a) \cdot (m \otimes n)$ with $\Delta : A \to A \otimes A$ a co-product.

A braided monoidal category also has a family of natural isomorphisms, $B_{xy} : x \otimes y \to y \otimes x$, subject to the hexagon equations.



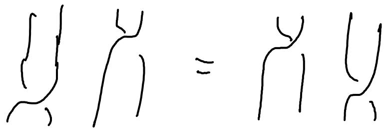
Having duals means having morphisms





Some discussion of locality/naturality.

Bifunctoriality of $\otimes$ implies locality in space:



Quantum invariants of tangles.

Parenthesized.

<u>Thm</u> The category of (framed, oriented) tangles in 3D is the free braided monoidal category on 1 object and its dual. "Tang"

<u>NB</u> Suppose A is a QTHA and $V \in Rep(A)$. Then by freeness, get a functor of braided monoidal categories on one object w/ duals:

Freyd & Yetter 1986

$$Z_{A,V} : Tang \longrightarrow Rep(A)$$
$$* \longrightarrow V$$

Aside: Universal invariant?

$$\begin{tikzcd}
Tang \arrow[rr, "?"] \arrow[dr, "Z_{A,V}"] \arrow[d, "Z_{B,W}"'] & & \Omega \arrow[dl] \arrow[dll, bend left] \\
Rep\,B & Rep\,A &
\end{tikzcd}$$

$$\begin{tikzcd}
PT \arrow[r, "Z"] \arrow[dr] & Proj(PT) \arrow[d, "T_{g,R}"] \\
& Rep(g), R
\end{tikzcd}$$

Objective: Given a Lie bi-Alg g, want a bi-alg H whose semi-classical limit is $U(g)$. I.e., H should be a topologically free algebra over $\mathbb{Q}[[\hbar]]$

which is a bi-alg, and s.t. $H/\hbar H \cong U(g)$.

Scheme: Need a co-alg structure δ_H on H whose limit is δ_g. Insert g into a QTLBA Dg. Look at $\mathrm{Rep}\, U(Dg)$. It is a braided monoidal category (Use "blackbox": given an associator $\bar{\Phi}$) Use the tensor product in $\mathrm{Rep}\, U_{Dg}$ to get Δ on Dg.